Stats 1 - June 2009 (specimen)

① a) i) 100/160

ii) $\frac{160-32}{160} = \frac{128}{160}$

iii) $\frac{32+60-18}{160} = \frac{74}{160}$

iv) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{30}{100}$

b) $P(C, R, S) = \frac{24}{160} \times \frac{56}{159} \times \frac{32}{158} = \frac{43008}{4019520}$

6 ways of arranging C, R, S

$\rightarrow 6 \times \frac{43008}{4019520} = \frac{1344}{20935} = 0.064198...$

② a) From calculator: $\Sigma x^2 = 30667$

$r = 0.84319...$

b) Strong, positive, linear correlation between length and widths of snakes.

c) See graph on mark scheme

d) i) G & D (closest to top-right of graph)

ii) $r \approx 0.5$ as there appears to be positive, but weaker linear correlation for males

③ $X \sim N(253, \sigma^2)$ $\sigma = 5$

a) i) $P(X < 250)$

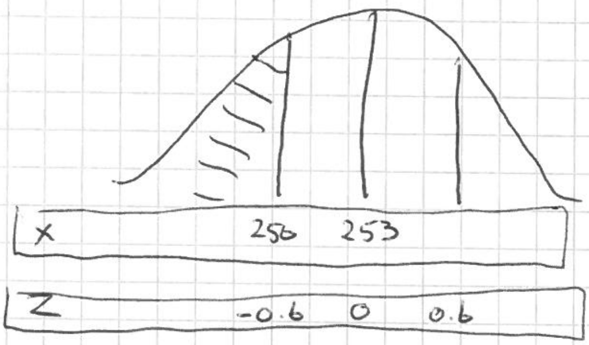
$= P\left(Z < \frac{250-253}{5}\right)$

$= P(Z < -0.6)$

$= P(Z > 0.6)$

$= 1 - P(Z < 0.6)$

$= 1 - 0.72575 = 0.27425$



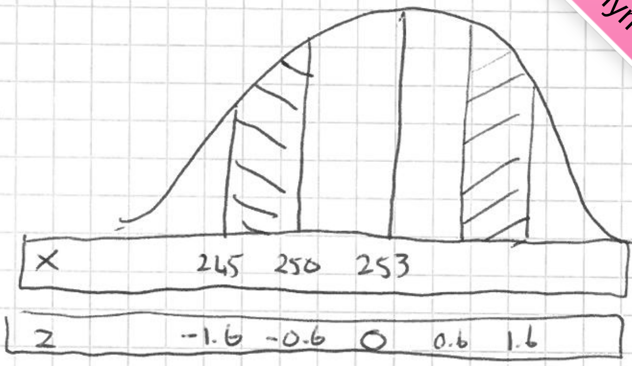
ii) $P(245 < x < 250)$

$= P\left(\frac{245-253}{5} < z < \frac{250-253}{5}\right)$

$= P(-1.6 < z < -0.6)$

$= P(0.6 < z < 1.6)$

(you can see they are same area on the diagram)



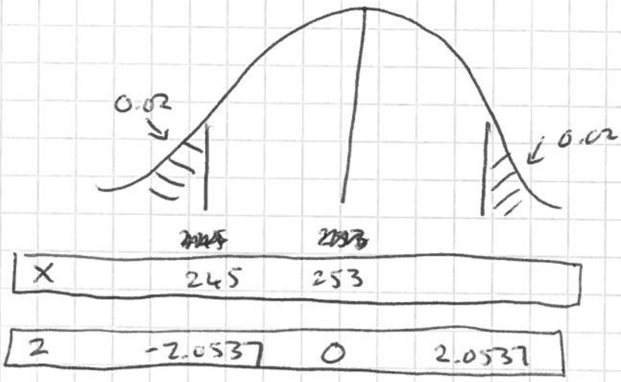
$= P(z < 1.6) - P(z < 0.6)$

$= 0.94520 - 0.72575 = 0.21945$

iii) $P(X = 245) = 0$

b) z value for 98%
$= 2.0537$

$\therefore$ z value for $x = 245$ must be -2.0537



Standardise:

$\frac{245 - 253}{\sigma} = -2.0537$

$245 - 253 = -2.0537\sigma$

$\sigma = \frac{-8}{-2.0537} = 3.8954...$

(4) a) From calculator: $\Sigma x^2 = 14975$

$a = 49.7982...$ (intercept)

$b = -0.54814...$ (gradient)

$\rightarrow y = 49.7982 - 0.54814x$

b) when $x = 0 \rightarrow y = 49.7982$

$= 50g$ (nearest g)

c) 13 weeks = 91 days $(7 \times 13) \rightarrow x = 91$

$\rightarrow y = 49.7982 - 0.54814(91)$

$= -0.08254g$

Negative weight implies the tablet has dissolved

$\therefore$ claim appears justified

⑤ a)

x	1	2	3	4	5	6	7
f	14	35	25	13	9	2	1
cum f	14	49	74	87	96	98	99

i) Median $= \frac{99+1}{2} =$ 50th value $=$ 3 children

LQ $= \frac{99+1}{4} =$ 25th value $=$ 2 children

UQ $= \frac{3(99+1)}{4} =$ 75th value $=$ 4 children

$\therefore$ IQR $= 4 - 2 = 2$ children

ii) From calculator: $\Sigma x^2 = 933$ $\Sigma x = 275$

Mean $= 2.777...$ ($\bar{x}$)

sd $= 1.31362...$ (s)

b) i) Σx still 275 $\rightarrow$ Mean $= \frac{275}{163} = 1.68711...$

ii) sd will increase due to increase in spread of data

iii) Mean $= 1.687$

New sd must be bigger than $1.313...$

Mean $- 2 \times$ sd $\rightarrow$ negative number of children!

And yet, for Normal distribution we would expect some data to be with 2 sds of the mean.

⑥ a) From Calculator: $\bar{x} = 47$ $s = 15$ $n = 10$

98% z multiplier (2 tailed) $= 2.3263$

98% CI for $\mu = \bar{x} \pm z \times \frac{s}{\sqrt{n}}$

$\rightarrow \mu = 47 \pm 2.3263 \times \frac{15}{\sqrt{10}}$

$\rightarrow \mu = 47 \pm 11.034...$

$\rightarrow (35.965, \quad 58.034)$

b) $\bar{Y} \sim N(108, \ ^{28^2}/_{40})$

$P(\bar{Y} > 120)$

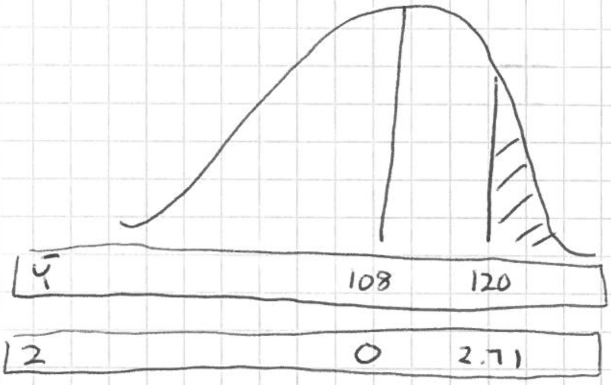
$= P\left(Z > \dfrac{120 - 108}{28/\sqrt{40}}\right)$

$= P(Z > 2.71)$

$= 1 - P(Z < 2.71)$

$= 1 - 0.9964$

$= 0.00336$



(4)

c) Yes. In b) we did not know if the population was normally distributed

7) a) $R \sim B(50, 0.15)$

i) $P(R < 10) = P(R \leq 9) = 0.7911$ (tables)

ii) $P(5 \leq R \leq 10)$

can be: 5, 6 ... 10

$\rightarrow P(R \leq 10) - P(R \leq 4)$

$= 0.8801 - 0.1121 = 0.768$

b) i) $F \sim B(22, 0.06)$

$P(F = 2) = {}^{22}C_2 \times 0.06^2 \times 0.94^{20} = 0.24125$

ii) $F \sim B(35, 0.06)$

$P(F \geq 1) = 1 - P(F = 0)$

$= 1 - {}^{35}C_0 \times 0.06^0 \times 0.94^{35}$

$= 1 - 0.11467...$

$= 0.8853$

iii) Correctly ! $\rightarrow$ $F \sim B(120, 0.94)$

$\boxed{\text{MEAN}} = np = 120 \times 0.94 = 112.8$

$\boxed{\text{VARIANCE}} = np(1-p) = 120 \times 0.94 \times 0.06 = 6.768$

iv) Means are the same (112.8)

$\therefore$ Agree probability she sorts a letter incorrectly is 0.06

But variances are very different. Much bigger for 20 batches

$\therefore$ Disagree that this is independent from letter to letter.